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Local commutativity and causality in interacting PP-wave string field theory

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Abstract: In this paper, we extend our previous study of causality and local commutativity of string fields in the pp-wave lightcone string field theory to include interaction. Contrary to the flat space case result of Lowe, Polchinski, Susskind, Thorlacius and Uglum, we found that the pp-wave interaction does not affect the local commutativity condition. Our results show that the pp-wave lightcone string field theory is not continuously connected with the flat space one. We also discuss the relation between the condition of local commutativity and causality. While the two notions are closely related in a point particle theory, their relation is less clear in string theory. We suggest that string local commutativity may be relevant for an operational defintion of causality using strings as probes.

Keywords: String Field Theory, Space-Time Symmetries, Black Holes.

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Contents

1. Introduction

Perturbative string theory provides a consistent quantization of gravity in the background of a flat spacetime. It is generally believed that nonperturbative string theory will lead to a fully consistent theory of quantum gravity and allows us to answer important questions concerning gravity such as the statistical origin of the blackhole entropy, the holographic nature of gravity, the blackhole information paradox and the fate of geometry at the very short distance scale etc. Over the years, many different frameworks have been proposed and studied. The most notably ones are the string field theory approach (lightcone string field theory [1]–[4], Witten covariant string field theory [5]), the Matrix model approach (BFSS matrix model [6], IKKT matrix model [7]) and the gauge/gravity approach of Maldacena [8]. Although it looks quite promising, we have not understood well enough of these different frameworks to obtain a controllable background independent nonperturbative formulation of string theory. It is therefore of great interest to understand better and to further develop these (and others, possibly new ones) formulations, and to understand the possible relations among them.

A common characteristic of these formulations is that they are formulated on objects that may not be directly related to the observables of interests (e.g. in AdS/CFT proposal, the bulk physics are dual to the quantities defined on the boundary; in string field theory, the string field is not directly related to the S-matrix). Thus it is possible that certain basic physical requirements which are expected to hold for any physical theory may not be so apparent and needed to be examined in more details. One of these is the requirement of causality.

Causality is easy to formulate in a point particle theory in terms of the propagation of light signal. It is also easy to implement in the classical theory, which amounts to the imposition of an appropriate boundary condition on the Green function. In a quantum field theory of point particle, causality is guaranteed by, among others, the condition of *local commutativity* (also called microscopic causality) of quantum fields [9]: quantum fields at spacelike separation (anti)commute. Now string is extended and is nonlocal, it is a priori not clear whether the theory is causal. Causality in the AdS/CFT correspondence was first studied by Polchinski, Susskind and Toumbas [10], who found that the causal propagation of classical wave packets in the AdS bulk leads to the prediction of extremely unusual degree of freedom in the dual gauge theory description. Causality in the bosonic lightcone string field theory formulation was first addressed by Martinec [11], who used the condition of local commutativity for the string fields to define a lightcone for the string.

String field theory holds the promise of giving a nonperturbative formulation of string theory beyond the usual first quantized form. This would be invaluable to the determination of the vacuum and to the formulation and discovery of the hidden string symmetry which give string theories their finiteness and other unusual properties, e.g. dualities. Lightcone string field theory has the advantage of being manifestly unitary and also that interactions are local in the lightcone time x^+. Hence a conventional canonical quantization can be performed and a second quantized operator formalism exists for the interacting theory.[1] The basic object in string field theory is the string field operator Φ that creates or annihilates string. Observables in the theory are expressed in terms of Φ. For example the bosonic part of the free string lightcone hamiltonian is given by

$$H_2 = \frac{1}{2} \int dp^+ \mathcal{D}^8 P(\sigma)\, \Phi^\dagger P^2 \Phi \,. \tag{1.1}$$

By employing a local commutativity condition on the lightcone string field, Martinec constructed the string lightcone for string theory in flat spacetime. Imposition of such a condition is quite natural since a string field can be written as a sum of all the component fields in the theory and it is reasonable to impose the condition of local commutativity on the component fields. The result of [11] gives a natural definition of what may be called the string lightcone. It is a natural extension of the usual particle lightcone and includes the higher string modes contribution. The result of [11] was later generalized in the framework of covariant string field theory [14]. The *same* result is obtained:

$$\text{the two string fields commute if } \int_0^\pi d\sigma (X^\mu(\sigma) - Y^\mu(\sigma))^2 > 0 \,. \tag{1.2}$$

This strongly suggests that concept of string local commutativity and the physics that one can extract from the string field commutator is gauge invariant.

These calculations were performed at the free string level. In the case of quantum field theory of point particles, interaction is local and the local commutativity condition (hence the lightcone) is not modified by the interaction. The corresponding situation in the case of string theory is less clear because, although string interact locally via splitting and jointing, the string itself is extended. Effects of string interaction on the local commutativity

[1] The canonical quantization of the covariant open bosonic string field theory was constructed in [12], and more recently by Erler and Gross [13]. The later paper also discussed the issues of locality and causality in the framework of the covariant string field theory.

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condition was first studied by Lowe, Susskind and Uglum [15]. These authors found that, due to the 3-string interaction, the string field commutator ceases to vanish outside the free string lightcone . This result does not necessarily imply that the string theory is acausal. As these authors argued, this is rather due to the nonlocal nature of the employed variable: the string field. Later Lowe, Polchinski, Susskind, Thorlacius and Uglum [16] argued that this nonlocal effect could lead to a break down of the usual nice slice assumption for the low energy theory in string theory and proposed that this may lead to a resolution of the blackhole information paradox.

A remark concerning the relation between local commutativity and causality is in order. In quantum field theory, one can show that the local (anti)commutativity of quantum fields (plus other conditions) leads to a causal S-matrix. In string theory, the connection between string local commutativity and causality displayed in the S-matrix is however much less clear. In fact from the above results of [15, 16], it was suggested that the reason for the nonvanishing contribution to the string field commutator was a effect of nonlocality of strings, rather than acausal behavior of the theory. The relation between string local commutativity and string causality was also questioned in the recent paper [13]. We find it reasonable and important to make distinction of the concept of *string local commutativity* and the usual concept of S-matrix causality. In this paper, we will be mainly concerned with the local commutativity condition and leave the important question of how it may be related to string causality for the future.

In the last two years, string theory in pp-wave background have been studied with immense interests, largely due to the remarkable proposal of Berenstein, Maldacena and Nastase (BMN) [17] which states that a sector of the SYM operators with large R-charge is dual to the IIB string theory on a pp-wave background. The string background consists of a plane wave metric ($d = 10$)

$$ds^2 = -2dx^+dx^- - \mu^2 \sum_{i=1}^{d-2} (x^i)^2 dx^+dx^+ + \sum_{i=1}^{d-2} dx^i dx^i \, , \tag{1.3}$$

together with a RR five-form field and a constant dilaton. The metric is invariant under SO(8) rotation of the transversal coordinates x^i. The RR background breaks it down to $\mathrm{SO}(4) \times \mathrm{SO}(4) \times Z_2$, where Z_2 exchanges the two SO(4) factors. Remarkably, even in the presence of curvature and a RR 5-form flux, the string theory is exactly solvable [18] in the lightcone gauge. It is therefore an interesting question to ask whether and how the causal structure of the theory is different from that in the flat case. Note that asymptotic state is not defined in the pp-wave string theory due to the increase of curvature at large distance, see (1.3). Thus the notion of a string S-matrix is not well defined and one cannot formulate the condition of causality in terms of the analyticity of the S-matrix. However the local commutativity condition is still well defined and appears to be more fundamental and universal. Therefore it is interesting to see whether the nonvanishing modification of the string field commutator persists in the pp-wave case. In our previous paper [19], we used the local commutativity condition of the free theory to define the string lightcone in the pp-wave string theory. We found that two strings in the pp-wave background has

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vanishing commutator if the condition eq.(2.10) is satisfied. This result is a modification of the flat space one. In particular it reduces smoothly to the flat space one in the flat space limit $\mu \to 0$ of the background. In this paper, we will study the effect of string interaction on the string local commutativity.

Important remarks on the 3-string vertex in the lightcone pp-wave string theory and its construction are in order. The 3-string vertex in the lightcone pp-wave string theory has been a subject of interest in the last two years. Motivated by the AdS/CFT duality, it is natural to suspect the existence of a correspondence (see [20]–[24] for some different proposals) between the three point functions of the BMN operators and the 3-string interaction vertex in the pp-wave lightcone string theory. Besides the applications, the string vertex is a string theoretic object that is of interest by itself. The bosonic part of the vertex can be determined uniquely by imposing the continuity condition on the embedding of the string worldsheet into spacetime [25, 26], or by using a path integral approach [27]. We will review the result in section 2 below. We note that this part of the vertex is continuous in the $\mu \to 0$ limit. The construction of the fermionic part of the vertex is however more subtle. In the flat case, the imposition of the kinematical symmetries fix the vertex up to a prefactor (which is a polynomial in p^+), which is then fixed by the imposition of the dynamical supersymmetries. However in the pp-wave case, the presence of the Z_2 symmetry leads to two possibilities in the choice of the fermionic vacuum and in the choice of the fermionic zero modes [29, 30]. Hence one can construct two inequivalent fermionic vertices that satisfy the fermionic kinematical constraints. Both can be completed supersymmetrically [25, 26, 28] and this had leaded to two different possible candidates for the lightcone 3-string vertex in the pp-wave background.

The main difference between these two approaches is in the symmetries that are respected by the vertex [27]. In the construction of [29, 30, 28], the Z_2 symmetry is realized explicitly. The resulting vertex is, however, not continuous in the $\mu \to 0$ limit. This is not surprising since the symmetry of the background is discretely different from the flat space one. In the construction of [25, 26] the continuity of interaction vertex in the $\mu \to 0$ limit is required instead. We will refer to this vertex as the "μ-continuous vertex". This condition seems natural since the string background is smooth in the $\mu \to 0$ limit. As it turns out, this requirement forces a different choice of the fermionic vacuum and the fermionic zero modes [29, 30] and hence a different vertex. There seems to be no first principle to fix the interaction directly within the lightcone framework. However, it turns out that the explicit form of the bosonic part of the vertex is sufficient for the purpose of our calculation. The reason is the following. Completing the vertex to the full supersymmetric case requires the inclusion of the fermionic vertex and the prefactor. However as we will explain in section 3, both of these won't modify our result since their contributions are sub-dominant. Therefore our results obtained using the bosonic string vertex (2.14) is unambiguous and applies to both the Z_2-invariant vertex and the μ-continuous vertex. Our main motivation was to use this to compute the effect of the pp-wave string interaction on the condition of local commutativity of the lightcone string fields and to see whether this effect is continuous in the $\mu \to 0$ limit.

Our result is surprising. We find that, unlike the flat case, the string field commutator does not receive any contribution for strings at causally disconnected region as determined by the free string lightcone (2.10). Thus the pp-wave string lightcone is unaffected by the string interaction! Intuitively this result could be understood since, compared to the flat case, the pp-wave string theory is more confined and more local due to the harmonic oscillator potential arises from the background. Since our result is for any $\mu \neq 0$, together with the results of [15], it means that the effect of the string interaction on the string field commutator is not smooth in the $\mu \to 0$ limit. We remark that it does not matter which vertices we use, any linear combination of the Z_2-invariant vertex and the μ-continuous vertex will give the same result. Our result shows that the tree level pp-wave string theory is not smoothly connected with the flat space theory even if the vertex is taken to be so. Since in any case quantities that one can compute from the theory are not necessary to be continuously connected with the corresponding quantities in the flat string theory, there is no compelling reason to require that the vertex to be continuously connected with the flat one.[2]

The organization of the paper is as follows. In section 2, we review the construction of the string field the pp-wave lightcone string field theory. We also construct the 3-string lightcone interaction in the number basis. In section 3, using the technique of contour deformation, we calculate the string field commutator in the presence of the 3-string interaction. We find that the string interaction does not modify the string field commutators in the pp-wave case. We discuss the results and its relevance and implications in the pp-wave/SYM correspondence . We end with a couple of further comments and discussions in section 4.

Note added: while the paper is being typed up, the paper [13] appeared on the archive which emphasized and examined the issue of locality and causality in string theory from the framework of covariant open string field theory, and partially overlaps with our work.

2. The 3-string interaction in PP-wave lightcone string field theory

In this section, we briefly review the construction of the lightcone string field in the pp-wave background [19] and construct the 3-string interaction vertex. We will leave α' arbitrary. Our final result is independent of α'.

2.1 Lightcone string field

The type-IIB pp-wave background consists of the metric (1.3), RR-form and a constant dilaton. Fixing the lightcone gauge $X^+(\sigma, \tau) = \tau$, the bosonic part of the string action is

$$S = \frac{1}{4\pi\alpha'} \int d\tau \int_0^{\pi|\alpha|} d\sigma [(\partial_\tau X^i)^2 - (\partial_\sigma X^i)^2 - \mu^2 (X^i)^2] , \tag{2.1}$$

[2] In fact, recently Dobashi and Yoneya [24] proposed that the pp-wave string vertex that is relevant for the holographic pp-wave/SYM correspondence is given by the equal weighted sum of the Z_2-invariant vertex and the μ-continuous vertex. As such, the vertex is neither continuous at $\mu = 0$ nor Z_2-symmetric. We refer the reader to the end of section 3 for more discussions.

where $\alpha = \alpha' p^+$. p^+ is the lightcone momentum and is positive (negative) for an outgoing(incoming) string. In the following analysis, we will focus on the bosonic part of the theory. Including the fermionic contribution will not modify our conclusion. For simplicity, we will often suppress the transverse indices i.

The mode expansions of the string coordinates and the conjugate momentum $P^i = \partial_\tau X^i / 2\pi\alpha'$ are given by

$$X^i(\sigma) = x_0^i + \sqrt{2}\sum_{l=1}^{\infty} x_l^i \cos\left(\frac{l\sigma}{\alpha}\right), \qquad P^i(\sigma) = \frac{1}{\pi|\alpha|}\left[p_0^i + \sqrt{2}\sum_{l=1}^{\infty} p_l^i \cos\left(\frac{l\sigma}{\alpha}\right)\right], \tag{2.2}$$

where $0 \le \sigma \le \pi|\alpha|$. In terms of the modes, the free string lightcone hamiltonian is

$$H = \frac{\alpha'}{|\alpha|}\sum_{l=0}^{\infty}\left(-\frac{\partial^2}{\partial x_l^2} + \frac{1}{4\alpha'^2}\omega_l^2 x_l^2\right), \qquad \omega_l = \sqrt{l^2 + (\mu\alpha)^2}\,. \tag{2.3}$$

This corresponds to a collection of simple harmonic oscillators with frequencies $\omega_l/|\alpha|$ and masses $|\alpha|/(2\alpha')$. The string field solves the Schrodinger equation $i\partial\Phi/\partial x^+ = H\Phi$ and is given by

$$\Phi(\tau, x^-, \vec{X}(\sigma)) = \int_0^\infty \frac{dp^+}{\sqrt{2\pi p^+}} \sum_{\{\vec{n}_l\}} A\left(p^+, \{\vec{n}_l\}\right) e^{-i\left(x^+ p^- + x^- p^+\right)} f_{\{\vec{n}_l\}}(\vec{x}_l) + \text{h.c.}\,, \tag{2.4}$$

where the coordinate space wave function is defined by

$$f_{\{\vec{n}_l\}}(\vec{x}_l) := \prod_{l=0}^{\infty} \varphi^l_{\{\vec{n}_l\}}(\vec{x}_l) \tag{2.5}$$

with

$$\varphi^l_{\{\vec{n}_l\}}(\vec{x}_l) := \prod_{i=1}^{d-2} H_{\{n_l^i\}}\left(\sqrt{\frac{\omega_l}{2\alpha'}}x_l^i\right) e^{-\omega_l (x_l^i)^2/4\alpha'} \sqrt{\frac{\sqrt{\omega_l/2\alpha'\pi}}{2^{n_l^i}(n_l^i!)}} \tag{2.6}$$

for each fixed l. Φ has eigenvalue

$$H = 2p^- = \frac{1}{|\alpha|}\sum_{i=1}^{d-2}\sum_{l=0}^{\infty} n_l^i \omega_l\,. \tag{2.7}$$

The Equal Time Commutation Relation for the string field is

$$\left[\Phi(x^+, x^-, \vec{X}(\sigma)),\, \Phi(x^+, y^-, \vec{Y}(\sigma))\right] = \delta(x^- - y^-)\prod_{i=1}^{d-2}\delta\left[X^i(\sigma) - Y^i(\sigma)\right]. \tag{2.8}$$

This give rises to the commutation relation for the string creation-annihilation operators

$$\left[A\left(p^+, \{n_l^i\}\right), A^\dagger(q^+, \{m_k^j\})\right] = p^+\delta(p^+ - q^+)\delta_{\{n_l^i\},\{m_k^j\}}\,. \tag{2.9}$$

By considering the unequal time commutator of the string fields $\Phi(x^+, x^-, \vec{X}(\sigma))$ and $\Phi(y^+, y^-, \vec{Y}(\sigma))$, one can define the string lightcone [11]. For the pp-wave case, we found that [19] the two string fields commute if[3]

$$\Delta x^- - \frac{\mu}{4\sin\left(\frac{\mu}{2}\Delta x^+\right)} \sum_{l=0}^{\infty} \left[(\vec{x}_l^2 + \vec{y}_l^2) \cos\left(\frac{\mu}{2}\Delta x^+\right) - 2\vec{x}_l \cdot \vec{y}_l \right] < 0 \,. \tag{2.10}$$

Here $\Delta x^+ := y^+ - x^+$, $\Delta x^- := y^- - x^-$. This result, when restricted to the zero mode sector, agrees precisely with the particle lightcone, either derived from the local commutativity condition of the point particle quantum field theory, or from the geodesic distance from (1.3). We remark that the above obtained zero-mode lightcone also agrees with the one obtained from the wave propagation point of view using the scalar [31], spinor and vector propagator [32] in the pp-wave background.

2.2 3-string interaction

Consider the interaction of 3 open strings with lightcone momentum p_{r}^+, $\mathsf{r} = 1, 2, 3$ respectively. The string vertex is required to satisfy all the kinematical and dynamical symmetries of the theory. This can be easily achieved by imposing the corresponding continuity condition. It is convenient to use a momentum representation of the string field. It is given by the Fourier transform with respect to y^- and $\vec{Y}$:

$$\tilde{\Phi}(x^+, \alpha, \vec{P}(\sigma)) := \int dy^- \mathcal{D}\vec{Y}(\sigma)\; e^{iy^- p^+ - i\int \vec{Y}\cdot\vec{P}} \Phi(x^+, y^-, \vec{Y}(\sigma)) \,. \tag{2.11}$$

In this basis, the bosonic part of the three-string interaction hamiltonian is given by

$$H_3 = g \int \prod_{\mathsf{r}=1}^{3} d\alpha_{\mathsf{r}} \mathcal{D}\vec{P}_{\mathsf{r}}(\sigma)\; \tilde{h}(\alpha_{\mathsf{r}}, \vec{P}_{\mathsf{r}}(\sigma)) \prod_{\mathsf{r}=1}^{3} \tilde{\Phi}(x^+, \alpha_{\mathsf{r}}, \vec{P}_{\mathsf{r}}(\sigma)) \,. \tag{2.12}$$

Here g is the string coupling, $\vec{P}_{\mathsf{r}}(\sigma)$ is the transverse momentum field of the r-th string, and The integration measure is defined in terms of the modes $\mathcal{D}\vec{Y}(\sigma) := \prod_{l=0}^{\infty} d\vec{y}_l$ and $\int \vec{Y}\cdot\vec{P} = \sum_{l=0}^{\infty} \vec{y}_l \cdot \vec{p}_l$. Without loss of generality, we assume $\alpha_1, \alpha_3 > 0, \alpha_2 < 0$, and the measure factor $\tilde{h}(\alpha_{\mathsf{r}}, \vec{P}_{\mathsf{r}}(\sigma))$ is given by[4]

$$\tilde{h}(\alpha_{\mathsf{r}}, \vec{P}_{\mathsf{r}}(\sigma)) = \delta\left(\sum \alpha_{\mathsf{r}}\right) \int \prod_{\mathsf{r}=1}^{3} \mathcal{D}\vec{Y}_{\mathsf{r}}(\sigma)\; e^{i\int \vec{P}_{\mathsf{r}}\cdot\vec{Y}_{\mathsf{r}}} \delta(\vec{Y}_2 - \vec{Y}_1 - \vec{Y}_3) \,, \tag{2.13}$$

which is basically a continuity condition.

To pass to the oscillator number basis, we substitute (2.11) and (2.4) and obtain

$$H_3 = g \int \prod_{\mathsf{r}=1}^{3} \frac{d\alpha_{\mathsf{r}}}{\sqrt{2\pi|\alpha_{\mathsf{r}}|}} \sum_{\{\vec{n}_{\mathsf{r},l}\}} \tilde{V}(\alpha_{\mathsf{r}}, \{\vec{n}_{\mathsf{r},l}\}) \prod_{\mathsf{r}=1}^{3} A\left(p_{\mathsf{r}}^+, \{\vec{n}_{\mathsf{r},l}\}\right) + \cdots \,, \tag{2.14}$$

[3]xHere we correct a typo in the final result of the paper [19]. μ there should be replaced by $\mu/2$.

[4]The string with α which is opposite in sign to the other two's has the widest transverse extension and hence the form of the delta-functional.

where the ... are terms of the form $AAA^\dagger, AA^\dagger A^\dagger, A^\dagger A^\dagger A^\dagger$. For the calculation to be performed below, only the AAA term is relevant. In (2.14), $\tilde{V}(\alpha_{\mathrm{r}}, \{\vec{n}_{\mathrm{r}l}\})$ is the 3-string vertex in the oscillator number basis. It is given by

$$\tilde{V}(\alpha_{\mathrm{r}}, \{\vec{n}_{\mathrm{r},l}\}) = \int \prod_{\mathrm{r}=1}^{3} \prod_{l=0}^{\infty} d\vec{p}_{\mathrm{r},l}\, \tilde{h}(\alpha_{\mathrm{r}}, \vec{P}_{\mathrm{r}}(\sigma)) \prod_{\mathrm{r}=1}^{3} \tilde{f}_{\{\vec{n}_{\mathrm{r},l}\}}(\vec{p}_{\mathrm{r},l})\,, \tag{2.15}$$

where

$$\tilde{f}_{\{\vec{n}_l\}}(\vec{p}_l) := \int \prod_{l=0}^{\infty} d\vec{y}_l\; e^{-i\sum_l \vec{y}_l\cdot\vec{p}_l} f_{\{\vec{n}_l\}}(\vec{y}_l) \tag{2.16}$$

is the momentum space wave function. We note that this part of the vertex is continuous in the $\mu \to 0$ limit.

The bosonic vertex (2.14) should be completed with the fermionic vertex and the prefactor in order to respect fully both the kinematical and the dynamical symmetries of the theory. As mentioned in the introduction, the construction of the fermionic part of the vertex is more subtle. It has been pointed out that due to the presence of the Z_2 symmetry, two possible choices in the fermionic vacuum and in the choice of the fermionic zero modes [29, 30] is allowed. Hence one can construct two inequivalent fermionic vertices that satisfy the fermionic kinematical constraints. Moreover both can be completed supersymmetrically [25, 26, 28] and this leads to two different possible candidates for the lightcone 3-string vertex in the pp-wave background. These two vertices have the same bosonic part, but different fermionic parts and different prefactors. However it is easy to see that for our computation, it is enough to use the above constructed bosonic vertex. One can easily see that [33] the fermionic contribution is sub-dominant in the $p^+ \to \infty$ limit, as compared to the exponentially growing behavior of the bosonic contribution (see (3.26) below). This is also the same for the contribution of the prefactor since the prefactor (in both cases) is a polynomial in p^+. Hence the bosonic contribution dominates in our computation.

3. String interaction and string field commutator

In this section, we investigate the effects of string interaction on the local commutativity condition of the string fields. Our analysis follows closely that of [16].

Consider the amplitude

$$M = {}_{\mathrm{H}}\langle 0|\,[\Phi_H(1), \Phi_H(2)]\,|3\rangle_{\mathrm{H}}\,, \tag{3.1}$$

of two string fields, denoted by 1 and 2, with a 3$^{\mathrm{rd}}$ spectator state. The spectator state is necessary for a possible non-zero contribution at first order in the string coupling constant g. The subscript H means that everything is in the hamiltonian picture. Passing to the interaction picture, we have

$$M = \langle 0; x_1^+|\Phi_I(1) U_I(x_1^+, x_2^+)\Phi_I(2)|3; x_2^+\rangle - \{1 \leftrightarrow 2\}\,, \tag{3.2}$$

where $U_I(x_1^+, x_2^+)$ is the time evolution operator in the interaction picture. In the leading order of string coupling, it is

$$U_I(x_1^+, x_2^+) = 1 + ig \int_{x_1^+}^{x_2^+} dx^+ H_3(x^+) + \cdots . \tag{3.3}$$

Hence up to first order in g we have

$$M = M^{(0)} + M^{(1)} , \tag{3.4}$$

where the zeroth order amplitude

$$M^{(0)} = \langle 0| \left[\Phi_I(1), \Phi_I(2)\right] |3\rangle \tag{3.5}$$

is a matrix element of the commutator of the two string fields, and

$$M^{(1)} = ig \int_{x_1^+}^{x_2^+} dx^+ \langle 0|\Phi_I(1) H_3(x^+)\Phi_I(2) + \Phi_I(2) H_3(x^+)\Phi_I(1) - \Phi_I(1)\Phi_I(2) H_3(x^+)|3\rangle . \tag{3.6}$$

For strings outside the string lightcone (2.10), we see immediately that $M^{(0)} = 0$. Any possible causality violations will come from a non-zero $M^{(1)}$.

Now, since H_3 is of the form Φ^3 and the string field of the form $\Phi \sim A + A^\dagger$, we can break the interaction vertex down to terms with equal number of creation and annihilation operators, $H_3 = H_{3\mathrm{aaa}} + H_{3\mathrm{aac}} + H_{3\mathrm{acc}} + H_{3\mathrm{ccc}}$. It is easy to see that unless the spectator state is a single string state of the form $|3\rangle = A^\dagger(p_3^+, \{\vec{n}_{3,l}\})|0\rangle$, $M^{(1)}$ will be identically zero. With this choice for the spectator state, we have

$$\begin{aligned} M^{(1)} = ig \int_{x_1^+}^{x_2^+} & dx^+ \langle 0|\Phi_{\mathrm{a}}(1) H_{3\mathrm{aac}}(x^+)\Phi_{\mathrm{c}}(2) + \Phi_{\mathrm{a}}(2) H_{3\mathrm{aac}}(x^+)\Phi_{\mathrm{c}}(1) - \\ & - \Phi_{\mathrm{a}}(1)\Phi_{\mathrm{a}}(2) H_{3\mathrm{acc}}(x^+)|3\rangle . \end{aligned} \tag{3.7}$$

Substituting (2.4) and (2.14), it is easy to obtain

$$\begin{aligned} M^{(1)} = ig \int_{x_1^+}^{x_2^+} d\tau \prod_{\mathrm{r}=1}^{2} \int_{-\infty}^{\infty} \frac{d\alpha_{\mathrm{r}}}{\sqrt{2\pi|\alpha_{\mathrm{r}}|}} F(\alpha_1, \alpha_2) \times \\ \times \sum_{\{\vec{n}_{1,l}\},\{\vec{n}_{2,l}\}} \left(f_{\{\vec{n}_{1,l}\}}(\vec{x}_{1,l}) f_{\{\vec{n}_{2,l}\}}(\vec{x}_{2,l}) \tilde{V}(1,2,3)\, e^{-i\tau \sum_{\mathrm{r}=1}^{3} p_{\mathrm{r}}^-} e^{i \sum_{\mathrm{r}=1}^{2} p_{\mathrm{r}}^- x_{\mathrm{r}}^+ + p_{\mathrm{r}}^+ x_{\mathrm{r}}^-} \right) \end{aligned} \tag{3.8}$$

where the function F is defined by

$$F(\alpha_1, \alpha_2) := \Theta(\alpha_1)\Theta(-\alpha_2) + \Theta(-\alpha_1)\Theta(\alpha_2) - \Theta(-\alpha_1)\Theta(-\alpha_2) . \tag{3.9}$$

Using (2.15) for the explicit expression of $\tilde{V}(1,2,3)$, and using sum rule for the Hermite polynomial, one can easily calculate the sum in the second line of (3.8) and obtain

$$\begin{aligned} M^{(1)} = ig \int_{x_1^+}^{x_2^+} & d\tau \int_{-\infty}^{\infty} \frac{d\alpha_1}{2\pi\sqrt{|\alpha_1(\alpha_1+\alpha_3)|}} \times \\ & \times \int \prod_{\mathrm{r}=1}^{3} \prod_{l=0}^{\infty} d\vec{y}_{\mathrm{r},l}\ \delta(\vec{Y}_2 - \vec{Y}_1 - \vec{Y}_3) J_{1,l}(\alpha_1, \vec{x}_{1,l}, \vec{y}_{1,l}) J_{2,l}(\alpha_2, \vec{x}_{2,l}, \vec{y}_{2,l}) \times \\ & \times f_{\{\vec{n}_{3,l}\}}(\vec{y}_{3,l}) e^{-i\tau p_3^-} e^{-ip_3^+ x_2^-} e^{-ip_1^+ \Delta x^-} . \end{aligned} \tag{3.10}$$

Here we have introduced the shorthand notation

$$J_{\mathsf{r},l}(\alpha_{\mathsf{r}},\vec{x}_{\mathsf{r},l},\vec{y}_{\mathsf{r},l}) := \left(\frac{\omega_{\mathsf{r},l}/2\alpha'}{\pi}\frac{1}{1-e^{-i\tau_{\mathsf{r}}\omega_{\mathsf{r},l}/|\alpha_{\mathsf{r}}|}}\right)^{(d-2)/2} \times$$
$$\times \exp\left\{\frac{\omega_{\mathsf{r},l}/2\alpha'}{2i\sin\left(\frac{\tau_{\mathsf{r}}\cdot\omega_{\mathsf{r},l}}{2|\alpha_{\mathsf{r}}|}\right)}\left[2\vec{x}_{\mathsf{r},l}\cdot\vec{y}_{\mathsf{r},l}-\left(\vec{x}^2_{\mathsf{r},l}+\vec{y}^2_{\mathsf{r},l}\right)\cos\left(\frac{\tau_{\mathsf{r}}\omega_{\mathsf{r},l}}{2|\alpha_{\mathsf{r}}|}\right)\right]\right\} \tag{3.11}$$

and $\Delta x^- := x_2^- - x_1^-$. The τ-dependence enters through $\tau_{\mathsf{r}} := \tau - x_{\mathsf{r}}^+$. For the kinematic situation we are considering here $(\alpha_1, \alpha_3 > 0, a_2 < 0)$, the delta-functional is

$$\delta(\vec{Y}_2 - \vec{Y}_3 - \vec{Y}_1) = \prod_{m=0}^{\infty}\delta\left(\vec{y}_{2,m} - \sum_{n=0}^{\infty}\left(\left|\frac{\alpha_3}{\alpha_2}\right|X^{(3)}_{mn}\vec{y}_{3,n} + \left|\frac{\alpha_1}{\alpha_2}\right|X^{(1)}_{mn}\vec{y}_{1,n}\right)\right). \tag{3.12}$$

The definition and properties of the matrices X are recalled in the appendix. Also for our case $F(\alpha_1, \alpha_2) = 1$.

To proceed further, one may write $M^{(1)}$ in the form

$$M^{(1)} = \int_{-\infty}^{\infty} d\alpha_1 K(\alpha_1)\, e^{-i\alpha_1\Delta x^-/\alpha'}\,, \tag{3.13}$$

where

$$K(\alpha_1) := \frac{ig}{2\pi\sqrt{|\alpha_1(\alpha_1+\alpha_3)|}}\int_{x_1^+}^{x_2^+} d\tau \int \prod_{\mathsf{r}=1}^{3}\prod_{l=0}^{\infty} d\vec{y}_{\mathsf{r},l}\delta(\vec{Y}_2 - \vec{Y}_1 - \vec{Y}_3) \times$$
$$\times \prod_{l=0}^{\infty} J_{1,l}(\alpha_1,\vec{x}_{1,l},\vec{y}_{1,l})J_{2,l}(\alpha_2,\vec{x}_{2,l},\vec{y}_{2,l})\cdot f_{\{\vec{n}_{3,l}\}}(\vec{y}_{3,l})e^{-i\tau p_3^-}e^{-ip_3^+x_2^-}\,. \tag{3.14}$$

Now let us focus our attention on the α_1 integral. As was done in [11, 19], we can write $M^{(1)}$ as

$$\int_0^{\infty} d\alpha_1 K(\alpha_1)e^{-i\alpha_1\Delta x^-/\alpha'} + \int_0^{\infty} d\alpha_1 K(-\alpha_1)e^{i\alpha_1\Delta x^-/\alpha'}\,. \tag{3.15}$$

Rotate the first integral by sending $\alpha_1 \to i\alpha_1$ and the second term by sending $\alpha_1 \to -i\alpha_1$. Then

$$M^{(1)} = i\int_0^{\infty} d\alpha_1 K(i\alpha_1)e^{\alpha_1\Delta x^-/\alpha'} - i\int_0^{\infty} d\alpha_1 K(i\alpha_1)e^{\alpha_1\Delta x^-/\alpha'}\,. \tag{3.16}$$

If each individual integral converges, the two terms cancel each other and hence $M^{(1)} = 0$. For that, we must examine the large α_1 behavior of $K(i\alpha_1)$. In [15, 16], it was found that for the flat case, the integral does not converge and the integral could picks up contribution from region outside the free string lightcone. Their result demonstrates the break down of local commutativity in the lightcone theory. However, it does not necessary mean that causality is violated. As the authors argued, this is rather due to the nonlocal nature of string itself. Below we will examine the same issue in the case of pp-wave string theory.

The above analysis was carried out for the general case with arbitrary string fields. It will be illuminating to consider a simplified situation where the 1st and 2nd string fields are taken to be the lowest component fields with:

$$\vec{n}_{1,l} = \vec{n}_{2,l} = \begin{cases} 0\,, & \text{when } l \geq 1\,, \\ \text{arbitrary}\,, & \text{when } l = 0\,. \end{cases} \tag{3.17}$$

The component field is obtained by integrating the string field with $\prod_{l=1}^{\infty} d\vec{x}_l \varphi^l_{\{\vec{n}_l\}}(\vec{x}_l)$. This gives

$$T(\tau, x^-, \vec{x}) = \int \frac{dp^+}{\sqrt{2\pi p^+}} \sum_{\vec{n}_0} a(p^+, \vec{n}_0) e^{-i(x^+p^- + x^-p^+)} \varphi^0_{\{\vec{n}_0\}}(\vec{x}) + \text{h.c.}\,, \tag{3.18}$$

where we have defined $a(p^+, \vec{n}_0) := A(p^+, \vec{n}_0, \{\vec{n}_{l\geq 1} = 0\})$ and in the following we often denote the zero mode $\vec{x}_0$ by $\vec{x}$ for simplicity. Furthermore, we restrict the 3rd string to be the following spectator state:

$$|3\rangle = A(p_3^+, \{\vec{n}_{3,l}\})|0\rangle\,, \qquad \text{with } \vec{n}_{3,l} = 0\,, \qquad \text{for all } l\,. \tag{3.19}$$

We note that $p_3^- = 0$.

Following the same procedures as above, it is easy to obtain (3.13) with $K(\alpha_1)$ now taking the form

$$\begin{aligned} K(\alpha_1) &= \frac{ig\, e^{-ip_3^+ x_2^-}}{2\pi\sqrt{|\alpha_1(\alpha_1+\alpha_3)|}} \int_{x_1^+}^{x_2^+} d\tau \int \prod_{\mathsf{r}=1}^{3} \prod_{l=0}^{\infty} d\vec{y}_{\mathsf{r},l}\, \delta(\vec{Y}_2 - \vec{Y}_1 - \vec{Y}_3) \\ &\quad \times J_{1,0}(\alpha_1, \vec{x}_{1,0}, \vec{y}_{1,0}) J_{2,0}(\alpha_2, \vec{x}_{2,0}, \vec{y}_{2,0}) \prod_{l=1}^{\infty} \varphi^l_{\{0\}}(\vec{y}_{1,l}) \varphi^l_{\{0\}}(\vec{y}_{2,l}) \cdot f_{\{0\}}(\vec{y}_{3,l})\,. \end{aligned} \tag{3.20}$$

We note that, compared with (3.14), the product $\prod_{l=1}^{\infty} J_{1,l}(\cdot) J_{2,l}(\cdot)$ in the second line there is replaced by $\prod_{l=1}^{\infty} \varphi^l_{\{0\}}(\vec{y}_{1,l}) \varphi^l_{\{0\}}(\vec{y}_{2,l})$ above due to the condition (3.17). Now we perform the contour rotation and focus on the integrals of the $\vec{y}$'s. Let us first intergate $d\vec{y}_{2,l}, l \geq 1$ using the nonzero mode delta functions. One can show that the resulting integral of $\vec{y}_{1,l}$ and $\vec{y}_{3,l}$, $l \geq 1$ is independent of the zero modes $\vec{y}_{1,0}$ and $\vec{y}_{3,0}$ in the large α_1 limit and so can be calculated easily. Next we integrate out $d\vec{y}_{3,0}$ using the zero mode delta function. Therefore in the large α_1 limit,

$$e^{ip_3^+ x_2^-} \frac{K(i\alpha_1)}{ig} \sim \int_{x_1^+}^{x_2^+} d\tau \int d\vec{y}_1 d\vec{y}_2\ \hat{J}_{1,0} \hat{J}_{2,0} \exp\left[+ \frac{\omega_{3,0}}{4\alpha'} \left(\left|\frac{\alpha_2}{\alpha_3}\right| \vec{y}_2 - \left|\frac{\alpha_1}{\alpha_3}\right| \vec{y}_1 \right)^2 \right] \tag{3.21}$$

up to an unimportant α_1-dependent proportional factor which is sub-dominant in large α_1 limit. Here we have denoted $\vec{y}_{\mathsf{r},0}$ by $\vec{y}_{\mathsf{r}}$ for simplicity. Also we have used the hat $\hat{}$ to denote the corresponding quantities with the substitution $\alpha_1 \to i\alpha_1$. For example, $\hat{\omega}_{1,l} = \sqrt{l^2 - \mu^2 \alpha_1^2}$ in $\hat{J}_{1,0}$. After the contour rotation and taking the large α_1 limit, we have

$$\hat{\alpha}_2 \sim -i\alpha_1\,, \qquad \text{and} \qquad \hat{\omega}_{1,l},\ \hat{\omega}_{2,l} \sim i\mu\alpha_1\,. \tag{3.22}$$

Now $\hat{J}_{\mathsf{r},0}$ takes the form

$$\hat{J}_{\mathsf{r},0} \sim \exp\left(-A_{\mathsf{r}}\left(\vec{x}_{\mathsf{r}}^2+\vec{y}_{\mathsf{r}}^2\right)+2B_{\mathsf{r}}\vec{x}_{\mathsf{r}}\cdot\vec{y}_{\mathsf{r}}\right) \tag{3.23}$$

with

$$A_{\mathsf{r}} = \frac{\mu\alpha_1/(2\alpha')}{2\tan(\frac{\mu}{2}|\tau_{\mathsf{r}}|)}\,, \qquad B_{\mathsf{r}} = \frac{\mu\alpha_1/(2\alpha')}{2\sin(\frac{\mu}{2}|\tau_{\mathsf{r}}|)} \tag{3.24}$$

and thus the $\vec{y}_1, \vec{y}_2$ integral takes the form $\int d\vec{y}_1 d\vec{y}_2 \exp(-\sum_{\mathsf{r},\mathsf{s}} N_{\mathsf{rs}}\vec{y}_{\mathsf{r}}\cdot\vec{y}_{\mathsf{s}} + \sum_{\mathsf{r}} \vec{S}_{\mathsf{r}}\cdot\vec{y}_{\mathsf{r}})$ and can be easily carried out. We obtain for the $\vec{y}_1, \vec{y}_2$ integral in (3.21),

$$\exp\left[-\frac{\mu\alpha_1/(2\alpha')}{2\tan(\frac{\mu}{2}\Delta x^+)}(\vec{x}_1^2+\vec{x}_2^2)+\frac{\mu\alpha_1/(2\alpha')}{\sin(\frac{\mu}{2}\Delta x^+)}\vec{x}_1\cdot\vec{x}_2\right] \tag{3.25}$$

in the leading large α_1 limit. It is remarkable that the various coefficients of $N_{\mathsf{rs}}, \vec{S}_{\mathsf{r}}$ combine to make the result (3.25) τ independent. Hence the τ integral in (3.21) can be calculated trivially. Finally we obtain for (3.16)

$$\int_0^\infty d\alpha_1 K(i\alpha_1)e^{\frac{\alpha_1}{\alpha'}\Delta x^-} \sim \int_0^\infty d\alpha_1 \exp\left[\frac{\alpha_1}{\alpha'}\left(\Delta x^- - \frac{\mu}{4\sin(\frac{\mu}{2}\Delta x^+)}\times\right.\right.$$
$$\left.\left.\times\left((\vec{x}_1^2+\vec{x}_2^2)\cos\left(\frac{\mu}{2}\Delta x^+\right)-2\vec{x}_1\cdot\vec{x}_2\right)\right)\right]. \tag{3.26}$$

The exponent of the integrand is precisely the tree level string lightcone (2.10) restricted to the zero modes. Thus we have shown that, unlike the flat case, the matrix element (3.1) does not receive contribution from region outside the free string lightcone.

As we remarked above, to be fully supersymmetric, the bosonic vertex has to be completed with the fermionic vertex and a prefactor which is needed for the preservation of the supersymmetries. Also the bosonic string field has to be replaced by the lightcone string superfield [4] so that we commute the matrix element of the commutator of two string superfields. Now the Grassmannian factor makes sub-dominant contributions to the contour-deformed integral in the limit $p^+ \to \infty$ and does not affect the convergence of the contour-deformed integral. This is also the case for the prefactor as it is a polynomial in p^+. Therefore we conclude that the commutator of the string fields is unaffected by the pp-wave string interaction. This is the main result of our paper.

Our result is surprising. Recall that in the flat case, the string field commutator was found to receive additional nonvanishing contribution [15, 16] from the interaction even if the two strings were outside the free theory string lightcone of each other. Since the pp-wave background and the bosonic part of the vertex are both continuous in the $\mu \to 0$ limit, one may natively thought that the pp-wave string field commutator should also receive additional contributions, at least in a neighborhood of $\mu = 0$. Our result shows that this is not the case and the matrix element (3.1) is discontinuous at $\mu = 0$. Technically the reason for the discontinuity is because the $\mu \to 0$ limit does not commute with the procedure of summing up the contributions from the infinite tower of string states. Our result shows that the UV behavior of the theory is sensitive to the IR parameter μ. This mixing of the IR and UV effects is similar to the IR/UV mixing in the noncommutative field theory [34].

Since the pp-wave lightcone string field theory is not continuously connected with the flat space string theory, there is no compelling reason to require that the 3-string vertex to be continuous at $\mu = 0$. What about the Z_2 symmetry? We would like to make the following remark. Without additional input, one cannot fix the form of the lightcone vertex uniquely from the supersymmetries alone. One may fix the form of the vertex by requiring the Z_2 symmetry. However there is a possibility that the symmetry is spontaneously broken. Since the pp-wave background is obtained from the AdS background by performing a Penrose limit, a reasonable possibility that may help to understand better the pp-wave string interaction is to try perform this limit carefully on the dynamics on the AdS side. This interesting idea has been pursued recently by Dobashi and Yoneya [24]. They propose that the pp-wave string vertex that is relevant for the holographic pp-wave/SYM correspondence is given by the equal weighted sum of the Z_2-invariant vertex and the μ-continuous vertex. It turns out that this particular combination coincides with the vertex proposed previously in [23]. These authors also provide some intuitive understanding of the role of each parts of the vertex: the Z_2-invariant vertex describes the "bare" interaction, while the μ-continuous vertex describes the mixing of the BMN operators. Thus according to this proposal, not only the continuity of μ is not maintained, also the Z_2 symmetry is broken due to the mixing effects. The breaking of the Z_2 symmetry has also been revealed in previous field theory calculations [35]. In principle, one can fix the form of the lightcone vertex by starting from the covariant Witten string field theory by doing the lightcone gauge fixing. To confirm this breaking from a more fundamental point of view will be very exciting.

4. Discussions and conclusions

Using the framework of lightcone string field theory, we computed the effect of string interaction on the string field commutator. We found that the string field commutator is unaffected by the pp-wave string interaction. This is the main result of our paper. Although our computation was performed at the first order in the string coupling, it is natural to conjecture that this remain the case to all order. Also for technical reason, we have restricted ourself to compute the matrix element for the special case (3.17) where the τ-integral is managable. The general case (3.13), (3.14) is more complicated. However we expect our conclusion remains the same.

In this paper, we used a lightcone gauge fixed string field theory framework. Technically, pp-wave string theory is tractable so far only in the lightcone gauge.[5] It is possible that the conclusion that one can drawn from the string fields commutator computation is gauge dependent. We don't have a proof that this is not the case. However it has been shown that in the case of free string in flat spacetime, the covariant string field theory and the lightcone string field theory gave the same result: demanding the vanishing of the

[5]Using the pure spinors formalism, Berkovits [36] has constructed an alternative covariant quantization for the pp-wave string theory. background. However the presence of a non–trivial background gives rise to a complicate world-sheet action, and explicit computations of amplitudes in this framework have not been done yet.

string field commutator give rises to the same string lightcone. This strongly suggests that the physics that one can extract from the string field commutator is indeed gauge invariant. We believe this is the same even when interaction is included, i.e. local commutativity of string fields is a gauge invariant physical concept. It will be very interesting to determine the string field commutator in the covariant string field theory [5] and see if one obtains the same nonlocal effect as in [15, 16]. The recently developed hamiltonian formalism [13] could be helpful in this respect.

If the string local commutativity is indeed gauge invariant, then what is its physical meaning? is it in some way related to the causality in string theory? It is important to understand what is the physical meaning of string local commutativity. Note that the usual operational definition of causality is based on the propagation of light signal and is a concept relevant for the low energy point particle theory. Imagine in a short distance scale where string scale is relevant. At this scale, we cannot ignore the nonlocal nature of the string probe. Obviously the usual definition of causality appears insufficient and we need a new operational definition. An important requirement is that this string form of causality should reduce to the usual form of causality in the low energy limit. We think the string local commutativity may have a chance to be related to this string form of causality.[6] It is an important question to explore.

In this paper, we examined the causality property in the interacting open string theory. The closed string case needs more discussion. An immediate question is what is the meaning of time in quantum gravity. It is generally believed that spacetime is a classical concept in a theory of quantum gravity, and the concept of spacetime loss its meaning when gravitational field is quantized. One may therefore question about the meaning of causality in the closed string theory. Although spacetime itself is not an observable in quantum gravity, it should still be useful to impose sensible condition, e.g. causality or local commutativity, in the formulation of the theory. For example we note the interesting suggestion of Teitelboim [37] who proposed to impose the condition of causality on the classical configurations (which makes senses) to be integrated in the path integral of gravity. In this sense, the study of causality become a valid question in closed string theory. And we propose that local commutativity could be a useful characterization for such in the closed string theory too.

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A. Fourier transform for open strings

Consider three open strings with lightcone momenta parametrized by α_r and with $\alpha_1, \alpha_3 >$

[6] Indeed if we use the string local commutativity to define a string lightcone, then this notion of causality indeed reduce to the ordinary one in the low energy limit since the effect found in [15, 16] vanishes as one takes $\alpha' \to 0$.

$0, \alpha_2 < 0$. The momentum of the strings are given by

$$P_{\mathsf{r}}(\sigma) = \frac{1}{\pi|\alpha|}\left[p_{\mathsf{r},0} + \sqrt{2}\sum_{l=1}^{\infty} p_{\mathsf{r},l}\cos\left(\frac{l\sigma}{\alpha}\right)\right], \tag{A.1}$$

with the coordinates of the three strings parametrized by

$$\begin{aligned} \sigma_1 &= \sigma\,, & 0 &\le \sigma \le \pi\alpha_1\,,\\ \sigma_3 &= \sigma - \pi\alpha_1\,, & \pi\alpha_1 &\le \sigma \le \pi(\alpha_1+\alpha_3)\,,\\ \sigma_2 &= -\sigma\,, & 0 &\le \sigma \le \pi(\alpha_1+\alpha_3)\,. \end{aligned} \tag{A.2}$$

The sum $P(\sigma) := \sum_{\mathsf{r}=1}^{3} P_{\mathsf{r}}(\sigma)$ admits the Fourier decomposition like (A.1) with

$$p_m = \sum_{\mathsf{r}=1}^{3}\sum_{n=0}^{\infty} X^{\mathsf{r}}_{mn} p_{\mathsf{r},n}\,, \qquad m \ge 0\,. \tag{A.3}$$

The matrix $X^{(2)}_{mn} = \delta_{mn}$ and for $\mathsf{r} = 1,3$

$$X^{(\mathsf{r})}_{mn} = \begin{cases} \tilde{X}^{(\mathsf{r})}_{mn}\,, & m > 0\,, n > 0\\ \frac{1}{\sqrt{2}}\tilde{X}^{(\mathsf{r})}_{m0}\,, & m > 0\\ 1\,, & m = 0 = n, \end{cases} \tag{A.4}$$

where for $m > 0, n \ge 0$,

$$\tilde{X}^{(1)}_{mn} := (-1)^n \frac{2m\beta}{\pi}\frac{\sin m\pi\beta}{m^2\beta^2 - n^2}\,, \qquad \tilde{X}^{(3)}_{mn} := \frac{2m(\beta+1)}{\pi}\frac{\sin m\pi\beta}{m^2(\beta+1)^2 - n^2}\,, \tag{A.5}$$

and $\beta = \alpha_1/\alpha_2$, $\beta + 1 = -\alpha_3/\alpha_2$.

The matrices $X^{(\mathsf{r})}$ satisfy the following identities

$$(X^{(\mathsf{r})T}X^{(\mathsf{s})})_{mn} = -\frac{\alpha_2}{\alpha_{\mathsf{r}}}\delta_{\mathsf{rs}}\,, \qquad \mathsf{r} = 1,3 \tag{A.6}$$

and

$$\sum_{\mathsf{r}=1}^{3} \alpha_{\mathsf{r}}(X^{(\mathsf{r})}X^{(\mathsf{s})T}) = 0\,. \tag{A.7}$$

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